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A MATHEMATICAL STUDY IN HISTORICAL PERSPECTIVE ON EARLY CHRONOGRAMS FROM CAMBODIA, VIETNAM AND INDONESIA

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Abstract

With four objectives, this paper conducts a mathematical study on the early chronograms from Cambodia, Vietnam and Indonesia in a historical perspective. First, the history and terminology of word-symbols from India to Southeast Asia is explored. This paper finds that the terms used for word-symbol in India are component-oriented while the term sengkala coined in Indonesia is application-oriented. Second, the reasons why the early chronogram-words have acquired the numerical-values they represent have been explored. Third, an insight is provided that the move, whether sinistral or dextral, used for writing word-symbol notation in India and chronogram in Southeast Asia had nothing to do with the maxim aṅkānāṃ vāmato gatiḥ. *Fourth, two points from the paper written by Apsari, Sariyasa, Junaidi, Tyaningsih and Gunawan are commented on. This paper suggests that the mathematical ideas employed in writing the chronograms be recognized as a history of mathematics. It acknowledges that the use of chronograms can make the teaching of mathematics culturally relevant.*

Keywords: *aṅkānāṃ vāmato gatiḥ*, *bhūtasaṅkhyā*, chronogram, and sengkala.

I. INTRODUCTION

The term chronogram is made up of two Greek words – *chronos* meaning time and *gramma* meaning writing. A chronogram is piece of writing, usually as an inscription, in which specific letters, interpreted as numerals, expresses a relevant year when rearranged or added. For example, a coin struck by Gustavus Adolphus includes the Latin inscription,

ChrIstVs DuX ergo trIVMphVs,

which means "Christ the Leader, therefore triumphant." This contains a chronogram. The capital letters, when added as Roman numerals, correspond to 1627 CE, which is the year the coin was minted (Rooney, 2013: 21). To clarify, C + I + V + D + X + I + V + M + V = 1627 because C means 100, I 1, V 5, D 500, X 10, and M 1000.

Unlike the above, a chronogram is regarded in Southeast Asian culture-areas as a phrase, usually in the form of an inscription but not necessarily, in which the year is assigned to number using words, not letters, which have connotations of numbers. The words that form a chronogram are usually called chronogram-words.

To decode the year, the chronogram is read according to the move followed to write it. The move can be dextral or sinistral. To write a required chronogram in the dextral move, its chronogram-words are arranged from left to right, starting at the highest place-value. They are decoded in the same order. To write a required chronogram in the sinistral move, its chronogram-words are arranged from right to left, starting at the highest place-value. But they are decoded in the reverse order; for example,

rttuśarena (i.e., *ṛtu-śara-ina*).

This refers to the year 1256 ŚaE for *ṛtu*, *śara* and *ina* have connotations of 6, 5 and 12 respectively. This chronogram is given in the treatise *Nāgara-Kĕrtāgama* (Skt. *Nāgara Kṛtāgama*), also referred to as the *Deśavarṇana* ("Description of the country"). This treatise is a *kakawin* (i.e., long descriptive poem) written in old Javanese as a worship scripture. It was written in honour of Hayam Wuruk, also called Rājasanagara, a king of the Majapahit Empire, by Mpu Prapañca in 1365 CE. The above chronogram refers to the birth year of this king. In fact, the first complete line of the stanza that contains it, when translated, reads that "[in] the *Śaka* year of season-arrow-sun, he was born to become a king" (Pigeaud, 1960: Vol. I, pp. xi-xii and 1.4, p. 3; Vol. III, p. 4; Pigeaud, 1963: 22).

Not every chronogram found in Southeast Asian culture-areas recorded in treatises. Indeed, the early chronograms have been found engraved on stones there. These culture-areas have a rich history of chronograms inscribed on stones from the seventh century to the fourteenth century (Annisa, 2011).

India exerted a great deal of cultural influence on the people of Southeast Asia. This still persists and can be easily traced in many social customs. One important influence from India is the chronogram system. Since A. W. Schlegel (1832 CE), James Princep (1834 CE), Eugène Jacquet (1835 CE), B. B. Datta and A. N. Singh (1935 CE), and J. Noorduyn (1993 CE) are among those scholars who have already discussed and proved that the system originally came from India through the then pan-India language Sanskrit, this paper will not attempt to discuss such an issue.

Apsari, Sariyasa, Junaidi, Tyaningsih and Gunawan are five Indonesian scholars of mathematics education. This quintet published a notable paper in 2021. The quintet writes in this paper that,

> "Some previous studies have only mentioned Candrasengkala [i.e. chronograms] as part of the historical background of certain places or phenomena. …, the discussion about the mathematical procedures in the numbers' arrangement of Candrasengkala was not given adequate proportion (Apsariet al., 2021: 2)."

I agree with the quintet not only in the sense they propose but also in the broader sense that the mathematical ideas employed in writing chronograms have not yet received enough

attention. To draw such attention, we will take early chronograms from Cambodia, Vietnam, and Indonesia and will do their case study from the point of view of the history of mathematics under four objectives, which I will let know in the next section.

For this case study, we will take only those chronograms that were inscribed on stones for the first time in each move in Cambodia, Vietnam, and Indonesia. Thus we will have six sets of chronograms. These three countries belong to Southeast Asian culture-areas. They are among the eleven countries into which Southeast Asia is currently divided. The rest include Laos, Malaysia, Myanmar, Singapore, Thailand, etc. Southeast Asia is bordered to the west by South Asia and the Bay of Bengal and to the south by Australia and the Indian Ocean.

II. METHOD

Data on early chronograms from Cambodia, Vietnam, and Indonesia were collected from published literature and then grouped. All this has been done in the next section. These data have been analyzed in section four qualitatively using the method of discussion with historical and cultural perspective under the following four objectives to gain a deeper understanding of their concepts, patterns and practices and to explore ideas related to them.

To better understand what these early chronograms from Cambodia, Vietnam, and Indonesia are, it is necessary to view them from a broader historical and terminological perspective. Therefore, the first objective of this paper is to explore the history and terminology of word-symbol system from India to Southeast Asia.

The second objective is to explore the reasons why the chronogram-words forming these early chronograms have acquired the numerical-values they represent. For the significance of the word-symbols "from Indian sources" and "other chronogram-words" referred to in this paper, see Appendix.

The maxim *aṅkānāṃ vāmato gatiḥ* is often discussed along with the word-symbol system. The chronogram written in the dextral move is said to be against the maxim. The year denoted by chronogram *kawihāji-pañca-pasāgi* in Inscription "Kebon-Kopi II" is often debated for the reason that this chronogram is not read according to the maxim (Prabowo et al., 2016: 5083-5084). Therefore, the third objective of this paper is to provide insight into whether the maxim and the moves are related.

As the fourth and last objective of this paper, I would like to comment on two points related to the quintet's paper: (1) recognition to chronograms as ethnomathematics, and (2) approach to cultural-based mathematics education through chronograms.

III. EARLY CHRONOGRAMS FROM CAMBODIA, VIETNAM AND INDONESIA

To study early chronograms inscribed in Cambodia, Vietnam, and Indonesia, we have compiled them in Table 1. We are able to see that there are six sets of chronograms in Table 1. Set one is the only set that has two chronograms. Everyone else has one.

Inscription "K. 13" engraved on a single face of a stele belonged to the Phnom Bayang Temple. It carries years of 526 ŚaE and 546 ŚaE noting the raising a brick wall to surround the divine foot of Lord Śiva on the platform of a mountain by Vidyādivindvanta and holy water being brought by him respectively (Majumdar, 1953: 8-10). Since both of the two chronograms, namely, *rasa-dasra-śara* and *ṛtu-vārinidhi-indriya*, coded for 526 ŚaE and 546 ŚaE respectively are lying in verse 11 itself, *rasa-dasra-śara* must be inferred to be chronologically as old as *ṛtu-vārinidhi-indriya*. Inscription "K. 323" is a double sided stele. It is also known as the Lolei (or Loley) inscription. Lolei Temple is an ancient brick temple group located east of Siem Reap. It is the northernmost temple of the Roluos group of temples (Barth, 1885: 219; Golzio, 2000: 21; Annisa, 2011: 57, 72 and 125).

Inscription "C. 74" engraved on two faces of a pillar belongs to Mỹ-so'n, which is a cluster of Hindu temples in central Vietnam. It bears the date of the year 653 ŚaE in which king Vikrāntavarman established here glory (*kīrtti*), i.e., an image of Lakṣmī (Majumdar, 1927, Book III: 38-41; Golzio, 2004: 26-28). Inscription "C. 30 B 3" is engraved on the right

door-pillar of the temple of Po-Nagar to the left. It carries the date of year 972 ŚaE in which king Śrī Parameśvara placed in the sanctuary of the divinity of Yapu-Nagara (i.e., an image representing Lord Śiva and his spouse Goddess Gaurī united as a single body) a vase inlaid with gold and has donated to that goddess for her worship an excellent diadem, a variegated waist-band, an umbrella decorated with peacock feathers, a vast silver canopy, and many other vases (Majumdar, 1927, Book III: 151-153; Golzio, 2004: 128-130).

Inscription "D. 4" was discovered among the ruins of the Śiva Temple complex located in the hamlet of Canggal on the plateau of the Wukir hill in Central Java. It is written in the early *Pallava* script. It carries date of 654 ŚaE when king Sañjaya established the Śivaliṅga there (Kern, 1917: 118; Chhabra, 1965: 45). Inscription "Kebon-Kopi II" was discovered in the hamlet of Pasir Muara in Bogor, West Java. It is written in old Malay language using the Kawi alphabet. It reads that,

//*Ini sa*bd*akalānda rākryañ juru pangāmbat i kawihāji pañca pasāgi marsāndeśa barpuliḥkan hāji sunda* (Bosch, 1941: 49)//

"This is the memorial [stone] ‹holding› the decree of Rākryan Juru Pangāmbat [i.e., Royal Hunter] ‹issued› in ‹the› [Śaka] ‹year› 854 and decreeing that the king of Suṇḍā is restored to his former state (Bosch, 1941: 50)."

According to F. D. K. Bosch, the fact that the edict was issued in the Malay language allows no other conclusion than that in the middle of the tenth century Suṇḍā was culturally and probably also politically subjugated to the power of the Sumatran Empire of Śrīvijaya (Bosch, 1941: 50).

The chronograms contained in these inscriptions are among the oldest not only with respect to their countries but also with respect to the move in which they are written. Datta and Singh referred to the chronograms contained in Inscription "K. 13" as the earliest (Datta and Singh, 1935: 59-60). This is why the historians of mathematics consider them as such (Sarma, 2009: 72).

But Annisa brings into our notice that Inscription "K. 756" written in Sanskrit and Khmer and discovered at Kompoṅ Spur contains the earliest chronogram in Cambodia. The portion of the inscription, which contains the chronogram, reads *khartugate śakendrasamaye* (Annisa, 2011: 121). The chronogram *kha-rtu-gati* denotes 460 ŚaE when decoded because *kha*, *rtu* (Skt. *ṛtu*) and *gati* denote 0, 6 and 4 respectively. The source of the reference she refers to for this is "Coedès, *Inscriptions du Cambodge* VI hal. 319."

Table 1: Early Chronograms from Cambodia, Vietnam and Indonesia

Country	Inscription	written in	Chronogram	Year decoded (in ŚaE)	Move (used to write)
Cambodia	K. 13	Sanskrit	*rasa-dasra-śara*	526	sinistral
			ṛtu-vārinidhi-indriya	546	sinistral
	K. 323	Sanskrit	*candra-indu-vasu-bhujā*	1182	dextral
Vietnam	C. 74	Sanskrit	*rāma-artha-ṣaṭ*	653	sinistral
	C. 30 B 3	Sanskrit	*velā-adri-nava*	972	dextral
Indonesia	D. 4	Sanskrit	*śruti-indriya-rasa*	654	sinistral
	Kebon-Kopi II	old Malay	*kawihāji-pañca-pasāgi*	854	dextral

There is one more chronogram which denotes an earlier year than 546 ŚaE (and 526 ŚaE). That chronogram is *kha-dvi-śara*. This is written in the sinistral move. It denotes 520 ŚaE. It is in Inscription "K. 151." The site of origin of this inscription is to the south-east of Roban Romas. It recounts the erection of a statue of Lord Kapilavāsudeva in 520 ŚaE during the reign of Bhavavarman I (Cœdés, 1943-46: 5-7).

The reason why I did not place *kha-rtu-gati* in Table 1 is that I did not attest it in the source of the reference (Cœdés, 1954: 319). The reasons why I have not included *kha-dvi-śara* in Table 1 are two; firstly, the inscription "recounts" and secondly, Annisa does not refer to this chronogram anywhere throughout her exhaustive study. Maybe my efforts to trace them are not enough. Therefore, I have decided to discuss them in this paper.

IV. RESULTS AND DISCUSSION

1. History and terminology from India to Southeast Asia

The use of words to denote numbers is found in India long before the commencement of the Common Era. The earliest instance of this kind is found in the *Śatapatha Brāhmaṇa*. The word used to denote 4 in it is *kṛta*. *Kṛta* has also been used in the *Taittirīya Brāhmaṇa* in the same numerical connotation. *Kṛta* being the past passive participle of the Sanskrit root *kṛ* (to do) means "done." It by extension indicates something "well done." *Kṛta*, *tretā* ([marked with] three), *dvāpara* (two-side), and *kali* (quarrel or misfortune) were the four throws of the Vedic dice game. They represented 4, 3, 2, and 1 respectively. Since *kali* represented 1, it was the losing throw. Since *kṛta* denoted 4, it was the "winning" or "well done" throw. Since *gāyatrī* and *jagatī* are the names of metres of 24 and 48 syllables respectively, *gāyatrī* is used in the *Kātyāyana Śrauta Sūtra* to represent 24 and *jagatī* is used in the *Lāṭyāyana Śrauta Sūtra* to represent 48 (Datta and Singh, 1935: 57-58; González, 1989: 196).

The term that Datta and Singh coined for such word-symbols is word-numeral. The title of the first chapter of their book *History of Hindu Mathematics* is "Notation." The title of the tenth section of this chapter is "Word-numerals." While explaining the word-numeral system in the beginning of this section, they write,

> "A system of expressing numbers by means of words arranged as in the place-value notation was developed and perfected in India in the early centuries of the Christian era. In this system the numerals are expressed by names of things, beings or concepts, which naturally or in accordance with the teaching of the *Śāstras*, connote numbers (Datta and Singh, 1935: 53)."

Their contemporary scholar H. R. Kapadia sums it up as follows.

> "A notation in which any word conveying the idea of a number is used is called a 'word-symbol notation.' This notation has not only been extremely well-known in India from ancient times but has also been extra-ordinarily popular (Kapadia, 1937: xxii)."

What I note from the above section of the book *History of Hindu Mathematics* is that *rasa, dasra*, and *śara* are word-numerals and *rasa-dasra-śara* is word-notation for Datta and Singh. They sometimes used "word-chronogram" in place of word-notation, but not necessarily in the sense of a number representing the year (Datta and Singh, 1935: 53-63). And what I understand from Kapadia's above definition is that *rasa, dasra*, and *śara* are word-symbols and *rasa-dasra-śara* is word-symbol notation for him. Here I have taken "*rasa*, *dasra*, and *śara*" as an example.

The terms we have seen so far are "chronogram-words" and "chronogram," "word-numeral" and "word-notation," and "word-symbol" and "word-symbol notation." Of them, I for my part have chosen the terms "word-symbol" and "word-symbol notation" to use in this paper. Apart from Kapadia, Anthony

Diller also employed them (Diller, 1996: 127). We have yet to see more terms.

The *Chandaḥsūtra* of Piṅgala, generally placed in the second century BCE, is the earliest text which employs word-symbols systematically and quite extensively, but without place-value (Sarma, 2009: 68-70). Datta and Singh write,

> "The principle of place-value seems to have been applied to the word-numerals between 200 B.C. and 300 A.D. The earliest instance of the use of the word-numerals with place-value in its current form is found in the *Agni-Purāṇa*, a work which belongs to the earliest centuries of the Christian era (Datta and Singh, 1935: 58-59)."

Here it must be noted that in the light of Bill M. Mak's research, it cannot be accepted, which some scholars have been accepting, that "word-symbols with place-value" occurred for the first time in Sphujidhvaja's *Yavanajātaka* (Mak, 2013: 11-16; Sarma, 2009: 70-71). Well, Sarma writes,

> "By the end of the fifth century the *Bhūtasaṅkhyā* [i.e., object-numeral] notation with place value was fully developed and firmly established, so much so that Varāhamihira's *Pañcasiddhāntikā* expresses nearly all the numerical quantities in the entire work with this notation. At the very outset, the work mentions the epoch of this work as *saptāśviveda*, [i.e., *sapta-aśvi-veda*] i.e., Śaka 427 (= AD 505/6) (Sarma, 2009: 71)."

Before we see how Varāhamihira expressed the number of intercalary months in the *Pañcasiddhāntikā*, I would like to say that very soon we will see the authenticity of the term *bhūtasaṅkhyā* for word-symbol. He expressed the number of intercalary months as *nava-vasu-guṇa-rasa-rasa* (= 66389) (Neugebauer and Pingree, 1970: v. 1.8, p. 28 and v. 1.14, p. 30). Both of these notations, i.e., *sapta-aśvi-veda* and *nava-vasu-guṇa-rasa-rasa*, are written in the sinistral move. They are each partial word-symbol notation because *sapta* and *nava* are the number-names of seven and nine in Sanskrit respectively.

From this time onwards, the word-symbol notation was employed in almost all the subsequent works on *siddhānta* (mathematical astronomy) and *gaṇita* (mathematics). Not only is the list of those works too long to summarize, but also the number of word-symbol notations in them is too large to summarize. Nevertheless, some examples are as follows. Brahmagupta (c. 628 CE) mentions 4305312000 in the *Brāhma-sphuṭa-siddhānta* as *khatra-ravi-guṇa-śara-kha-dahana-samudra* (Dvivedin, 1902: v. 1.53, p. 17). Vaṭeśvara (904 CE) expresses in the *Vaṭeśvara-Siddhānta* one *mahāyuga* to be equal to "*danta-abdhi* multiplied by one *ayuta*" (= 4320000) *sauravarṣas* (Shukla, 1986: 3). Bhāskara II (born in 1114 CE and died after 1183 CE) gives the subtle value for π as *bha-nanda-agni* (= 3927) divided by *kha-bāṇa-sūrya* (= 1250) (Limaye, 2019: 403). In the *Gaṇita-sāra-kaumudī* which Ṭhakkura Pherū (c. 1315) composed in Apabhraṃśa Prakrit, he asks to put "*juga*, *gaha*, *loyaṇa*; *hara-nayaṇa*, *iṃdiya*, *muṇi*; *aṭṭaha*, *sasi*, *rasa*" to form a magic square of order 3 (SaKHYa, 2009: v. 4.45, pp. 31, 80, 172). It may here be noted that *aṭṭaha* (Skt. *aṣṭa*) is the number-name of eight in Prakrit.

The use of the word-symbols was not limited to the works on mathematics and astronomy. Rather, the word-symbols were employed equally and frequently in the works on other disciplines of knowledge. In his *Trilokasāra*, Nemicandra (c. 981), who belonged to the canonical class of the Jaina school of Indian mathematics, expresses in Prakrit the number of mustard seeds obtained in the first unsteady cylindrical pit resembling the Jambūdvīpa as

vidhu-ṇidhi-ṇaga-ṇava-ravi-ṇabha-ṇidhi-ṇayaṇa-bala-ddhi-ṇidhi-khara-hatthī

with 31 zeros (= 19791209299968 × 10^{31} when read in the dextral move) where *ṇava* is the number-name of nine in Prakrit (Mukhtara and Patni, 1975: v. 21, p. 29; Jadhav, 2017: 322). Nāgavarma of the late 10th century used various word-symbols in his *Kannaḍa Chandassu*

("Canarese Prosody") (Jadhav, 2019: 4-8). Even horoscopes used word-symbols (Sarma, 2021: 172-173).

There is also no dearth of instances of the use of word-symbol system for coding the year in Indian epigraphic records. The earliest epigraphic evidence of this system is an inscription of Pratihāra Vatsarāja. This inscription is written in Sanskrit and its find-spot is unknown. It reads that the blueprint was duly conceived by the architects on the tenth day of the bright fortnight of the month Vaiśākha in the *Śaka* year coded as *muni-śaśi-naga* (= 717) for the construction, by the king Gallaka, of a temple for the goddess Caṇḍikā (Ramesh and Tewari, 1975-76: vv. 20-21, pp. 51 and 57; Acharya, 2001: 181).

Sarma observes that,

> "As the word-numerals began to be employed widely, attempts were made to prepare metrical lists or lexica of these words so that the beginners can learn them by heart (Sarma, 2009: 72)."

Examples are numerous to substantiate his observation. We would like to take only two. Rājāditya of the twelfth century provides a list of nearly 242 word-symbols denoting 1 to 9 plus 0 in his *Vyavahāra-gaṇita* (Jadhav, 2018: 53-66). Nijaguṇa Śivayogī, flourished sometime between 1250 CE and 1655 CE, gives a list of fifty-nine word-symbols denoting from 1 to 9 plus zero in his *Viveka-Cintāmaṇi*. Interestingly, the business community also needed to be aware of this list, he says (Jadhav, 2023: 14-15). The *Vyavahāra-gaṇita* is a work on business-mathematics while the *Viveka-Cintāmaṇi* is an encyclopaedia. Both of them are composed in Kannaḍa.

Above we have seen the terms coined by some modern scholars for the symbolic words. Some other modern scholars, although they did not coin any specific term, have discussed word-symbols under different titles. Employing the term word-numeral, D. C. Sircar puts forward "numbers expressed by words" as a title under which he discusses them (Sircar, 1965: 228-229). In keeping with the sense specifically intended for chronology, S. K. Acharya discusses word-symbol notations under the title "chronogrammatic system of notation" and calls them "chronogrammatic expression" (Acharya, 2001: 179-180).

Now let us see what terms the savants used for word-symbols. At the beginning of his *Gaṇita-sāra-saṅgraha*, Mahāvīra (c. 850 CE) gives a versified list of word-symbols denoting from 1 to 9 plus 0 under the heading *saṅkhyā-sañjñā* ("symbols for numbers") (Rangacharya, 1912: vv. 1.53-62, pp. 6-7). The heading under which Nijaguṇa Śivayogī gives a list of word-symbols is *gaṇita-sañjñe* (Skt. *gaṇita-sañjñās*, numeric-indicatives) (Jadhav, 2023: 18). S. R. Sarma notes that Āryabhaṭa II (c. 950 CE) calls word-symbols *prasiddha-sañjñās* ("well-known symbols") (Sarma, 2009: 68; Dvivedin, 1995: v. 15.1, p. 143).

Besides word-numeral, another term for the symbolic words, which is also very popular among modern scholars including David Pingree, K. V. Sarma, etc., is *bhūtasaṅkhyā* ("object-numeral"). This term has historical authenticity, which lies in the fact that it was used by a renowned savant whose name was Sundararāja. He employed it at the end of the fifteenth or the beginning of the sixteenth century in his commentary *Laghuprakāśikā* on the *Vākyakaraṇa*, but how old it is is not known. Sarma writes that no other Sanskrit treatise seems to have used it in this sense (Pingree, 1981: 1; Sarma, 2009: 67).

It is for the above reason that K. V. Sarma considers "object-numeral" to be a more exact term than "word-numeral," which he mentions in his definition of object-numeral as follows.

> "The word-numeral, or, to be more exact, 'object-numeral' (*bhūta-saṅkhyā*) refers to a system of numerical notation where a word denoting an object, concept, idea or group implies, besides its normal meaning, a number also by dint of meaning, convention or usage and, for that reason that word is used to denote the number in question (Sarma, 2003: 38)."

Annisa, followed by Prabowo et al., writes in the glossary of her thesis on "Chronograms in Indonesia, Vietnam and Cambodia from 7th century A.D. to 14th century A.D.: Archaeology and epigraphy approach" that the term which is used in Vietnam and Cambodia to describe the method of assigning years to numbers using words that have the connotation of numbers is *bhūtasaṅkhyā* (Annisa, 2011: 151; Prabowo et al. 2016: 5080). This information may prove to be of great historical importance in determining how old the term *bhūtasaṅkhyā* is. Therefore, it needs to first verify the information properly and then investigate when and where this term was first used in Vietnam and Cambodia.

Before we move on to Indonesia, we would like to see what term Kim Plofker used for the word-symbol system, how Medha Limaye used "word-numeral" and "object-numeral" together, and what the Kali-chronogram was in India. Plofker writes,

> "A different representation of decimal place value is revealed by a verbal notation called by medieval authors *bhūta-saṅkhyā* or 'object-numbers,' here designated the 'concrete number system' (Plofker, 2009: 47)."

But she has not given any reason for this new designation "concrete number system." First S. R. Sarma and then Annisa have also made a similar comment (Sarma, 2009: 68; Annisa, 2011: 3).

While commenting upon the use of word-symbols in the *Līlāvatī*, Limaye writes,

> "One may find verses [in the *Līlāvatī*] employing both the standard word numerals and the object numerals. For instances, in the verse giving the [gross] value of π $\left[=\frac{22}{7}\right]$,…, the author [i.e., Bhāskara II] has used numeral *dvāviṃśati* for twenty-two and the object numeral *śail* for seven (Limaye, 2019: 403)."

In fact, she employs "standard word-numeral" here for *dvāviṃśati*, the number-name of twenty-two in Sanskrit, by putting "standard" before "word-numeral."

The number of days that have elapsed since the beginning of a chosen era is called *ahargaṇa*. When the chosen epoch is Kali Era, it is called *Kali-ahargaṇa*. The present Kali Era commenced at sunrise on Friday, February 18, 3102 BCE. *Kali-ahargaṇas* were often used in South India, especially in the Kerala school of Indian mathematics, to date important events. They were written using the usual *kaṭapayādi* system. Modern scholars call the notation written in this manner Kali-chronogram. For example, Nārāyaṇa Bhaṭṭatiri marked the date of death of his teacher Acyuta Piṣāraṭi as *vidyātmāsvarasarpat* (meaning, when translated, "that learned soul [i.e., Acyuta] passed to heaven"), which is decoded as Kali-day 1724514 (which corresponds to the Julian date Friday, August 4, 1620). The *kaṭapayādi* system was widely popular in India. Unlike the word-symbol system, the digits in this system are usually represented by the consonants of the Sanskrit alphabet (Gupta, 2011: 518; Sarma, 2012: 38-61).

Sengkala or *sangkala* is a Javanese term in Indonesia for chronogram. "It was called," writes Teeuw (1998: 369), "*śākakāla* in old Javanese." *Śākakāla*, when written with the fullest loyalty to Sanskrit, must be *śakakāla*, which must have been obtained by joining two terms *Śaka* and *kāla* (time). On this basis, we can say that *sengkala* is a time recorded or expressed in *Śaka* Era. For example, "*ṛtu-śara-ina*" and the chronograms written in both of the Inscription "D. 4" and Inscription "Kebon-Kopi II" are *sengkalas*.

In Javanese literature we often read the term *candrasengkala*. Jérôme Petit writes,

> "[Thomas Stamford] Raffles explains erroneously that the designation *Candrasaṅkala* means 'Reflections of royal times,' which [Eugène] Jacquet corrects to 'Names of numbers beginning with the word *candra* ‹which stands for 1›' (Petit, 2009: 33)."

Rather the fact is that *candrasengkala* is one of the two forms of *sengkala*. On the basis of calendar being used *sengkala* is divided into

two forms: *suryasengkala* and *candrasengkala*. *Suryasengkala* uses solar calendar while *candrasengkala* uses lunar calendar. *Suryasengkala* refers to the *Śaka* calendar or the *Masehi* calendar (i.e., Current Era). *Candrasengkala* refers to the Javanese calendar or the Islamic calendar. Prior to the current system of Javanese calendar, which was inaugurated by Sultan Agung of Mataram in 1633 CE, the Javanese had used the *Śaka* calendar.

On the basis of contents being used *sengkala* is again divided into two forms: verbal *sengkala* and non-verbal *sengkala*. The verbal *sengkala* is called *sengkala lamba*. For example, "*sirna ilang kertaning bumi*" (= "The wealth of earth disappeared and diminished") corresponds to the *Śaka* year 1400 when read in the sinistral move, the date of the fall of the Majapahit Empire. Thus *sengkala lamba* has three elements – words, figures, and year. The words denote the corresponding figures and those figures together represent year when read in the move in which the *sengkala lamba* is written. The non-verbal *sengkala* is called *sengkala memet*. It is formed of images, particularly in three dimensional shapes on the temple, on the building, on the gateway and the like. Those images or their combination definitely represents a series of words. To obtain the year which the *sengkala memet* represents, it is changed into its corresponding *sengkala lamba*. An example can be observed along the Hall of Royal Novices, at the Sultanate Court of Ngayogyakarta Hadiningrat in Yogyakarta, where two dragons are arranged back-to-back in a painting, with one facing left and the other right, with their tails entwined in the middle. The artifact is translated as a sentence, becoming "*dwi naga rasa tunggal*" which means that "two dragons share feelings" or "two snakes unite in the same feeling," which represents the year 1682 AJ when read in the sinistral move. This is the year when the building was erected. It may here be noted that *dwi* (Skt. *dvi*) is the number-name of two.

This artifact is an example of *candrasengkala memet* while "*sirna ilang kertaning bumi*" is an example of *suryasengkala lamba*. For the above details on *sengkala*, see (Bratakesawa et al., 1980: 21-29, 107-109; Wijayatno 2003; Prabowo et al., 2016: 5079-5082; Listya and Pratama, 2019: 35-39).

According to Annisa, non-verbal *sengkala* was not found in any other country of Southeast Asia until now (Annisa, 2011: x). It would be interesting to know how old the term *sengkala* is, which I have not been able to find.

Sengkala and Kali-chronogram are somewhat similar in the sense that the word-symbol system in conjunction with the Śaka Era acquired a distinct identity as *Sengkala* in Indonesia and the *kaṭapayādi* system with the Kali Era gained the same identity in India itself, which modern scholars call *Kali-ahargaṇa* or Kali-chronogram.

Interestingly, the earliest inscription where chronogram was employed in word-symbol notation was found in Cambodia and not in India. This is said on the basis of "K. 13" (and also from "K. 756" and "K. 151") and the inscription of Pratihāra Vatsarāja. Possibly taking into account such observation from the Kaḍaba plates and the Dholpur Inscription of the *Śaka* years equivalent to 813 CE and 842 CE respectively, G. R. Kaye asserts that,

> "A [word-symbol] notation that became extraordinarily popular in India …was introduced about the ninth century, possibly from the East (Kaye, 1915: 31)."

But Datta and Singh write that,

> "[Such an assertion] shows his [i.e., Kaye's] ignorance of Indian mathematical works, or is a deliberate misrepresentation (Datta and Singh, 1935: 59)."

The Kaḍaba plates and the Dholpur Inscription were the earliest Indian epigraphic evidences with chronograms in word-symbol notation in the time of Datta and Singh, but the Pratihāra Vatsarāja Inscription is the earliest in our time.

We are able to see that Indian savants used different terms for word-symbol. They were not unanimous on any single term. Despite this, their terms have two components in common.

Firstly, all of them have *saṅkhyā* (numerals) or *gaṇita* (mathematics) in their names. Secondly, they all have *sañjñā* (symbol or indicative) or *bhūta* (object) in their names. Those well-known symbols or indicatives or objects could be actual or abstract. For example, *śaśi* (moon) is actual while *guṇa* (quality) is abstract. On this basis we can say that their terms are component-oriented. On the other hand, the term *sengkala* ("time in *Śaka* [Era]") from Javanese culture-areas is application-oriented as expressing the year in word-symbol notation is one of the areas of application of the word-symbol system. From its origin, the word-symbol system in India was not confined to a specific area of its application. On the other hand, from its entry into Southeast Asia from India, the word-symbol system remained confined to its use as chronogram. Commendable is that later, the Javanese took this area as an art form and developed it to such a high level that they created *sengkala memet*.

2. Early chronogram-words and their numerical significance

I have listed the chronogram-words found in the early chronograms from Cambodia, Vietnam and Indonesia in Table 2. They are nineteen in total. *Kha-dvi-śara* in Inscription "K. 151", *rāma-artha-ṣaṭ* in Inscription "C. 74", *velā-adri-nava* in Inscription "C. 30 B 3", and *kawihāji-pañca-pasāgi* in Inscription "Kebon-Kopi II" are such chronograms that have not been created using only word-symbols because *dvi*, *ṣaṭ*, *nava*, and *pañca* are the number-names of two, six, nine, and five in Sanskrit respectively.

Of these nineteen word-symbols, seventeen are in Sanskrit. The remaining two, namely, *kawihāji* and *pasāgi*, are written in old Malaya. These two seem to have been added to the then domain of Sanskrit word-symbols as new vernacular words. Or they seem to have adapted to the local language after they had pervaded in their original forms from Indian culture-areas into Southeast Asian culture-areas. O. W. Wolters puts the concept of localization as follows.

> "I believe," writes he, "unless there is convincing evidence to the contrary, that Indian materials tended to be fractured and restated and therefore drained of their original significance by a process which I shall refer to as 'localization' (Wolters, 1982: 52)."

Rasa, *dasra*, *śara*, *ṛtu*, *vārinidhi*, and *indriya* are those six word-symbols that formed two chronograms in Inscription "K. 13" (dated 546 ŚaE). These word-symbols except *vārinidhi* had already been used in India for forming word-symbol notations in the *Pañcasiddhāntikā* (427 ŚaE) of Varāhamihira. The same is true with *kha* and *ṛtu*, used in forming the chronogram in Inscription "K. 756", and with *kha* and *śara*, used in forming the chronogram in Inscription "K. 151". However, not *vārinidhi*, but its synonym *jaladhi* was employed in the *Pañcasiddhāntikā* (Neugebauer and Pingree, 1970: 185). All this shows that the earliest chronogram, whether in Inscription "K. 13" or in Inscription "K. 756" or in Inscription "K. 151", in Southeast Asia used those Indian word-symbols that had already been used in India in 427 ŚaE.

Now we will explore the significance of the above nineteen word-symbols so that we can know why they have acquired the numerical values which they represent.

Chronogram-words denoting one

Anything that is marked unique denotes one. Moon is the only one satellite of Earth. Therefore, *candra* (moon) and its synonym *indu* (moon) each refer to one.

Chronogram-words denoting two

According to the *Mahābhārata* and the *Viṣṇu Purāṇa*, Saṃjñā, the wife of Vivasvān (the Sun), was practicing austerities in the form of a mare (*aśvinī*). Vivasvān went to her disguised as a horse (*aśva*). Twin sons, Nāsatya and Dasra, were born to them. They are known as Aśvinīkumāra, Aśvi, Dasra, Aśva, etc. They are always seen as a pair (Mani, 1975: 69 and 680; Cappeller, 1891: 221). Hence, *dasra* is used to denote two. Body parts in pairs always

represent two. This is why *bhujā* or *bhuja* (arm) denotes two.

Velā has many connotations including time, either bank of the sea, tide, opportunity, etc. (Cappeller, 1891: 523). Barth supposes that *velā* is taken here in the sense of "tide" (Barth, 1885: 93). The reason, which I think, may be that tide, the periodic rise and fall of the waters of the ocean, is of two types – ebb and flow. As far as I searched, I could not find the use of the term *velā* as two in the ancient Indian literature on astronomy and mathematics. Even its use as two is not found in any other chronogram in Southeast Asia (Annisa, 2011).

Chronogram-word denoting three

According to the Hindu epics, Paraśurāma, Rāmacandra and Balarāma are three Rāmas. Paraśurāma, the son of Jamadagni, is always seen with his battle-axe. Rāmacandra, the hero of the epic *Rāmāyaṇa*, was the son of Daśaratha. Balarāma, the elder brother of Kṛṣṇa, is known as the patron of agriculture and is always seen as armed with a plough-share (Harshananda, 2008: Vol. 3, p. 14; Limaye, 2019:397). For this reason, Rāma refers to three.

Chronogram-words denoting four

Since *Arvāvat*, *Parāvat*, *Sarasvat*, and *Śaryaṇāvat* are the four seas mentioned in the *Ṛgveda* (Bhargava, 1964: 1-23), *vārinidhi* (sea) denotes four. The *Ṛgveda* ('Praise-knowledge'), the *Yajurveda* ('Sacrifice-knowledge'), the *Sāmaveda* ('Chant-knowledge'), and the *Atharvaveda* ('Priest-knowledge') are four *Vedas*. Since the *Vedas* are *Śruti* ("that which is heard"), *śruti* refers to four. *Pasāgi* is, according to Bosch, unusual as a word-symbol denoting number, but cannot denote anything other than four (Bosch, 1941: 50). *Pasāgi* (or *pasagi*) means square (Shellabear, 1912: 4 and 119; Egner, 1920: 77). The square shape always has four sides. This may have been the reason why Bosch decoded *pasāgi* as four. According to some *Purāṇas*, there are four paths (*gatis*) for a human being to approach God –sacrifice (*yajña*), austerity (*tapa*), renunciation (*karmasaṃnyāsa*), and spiritual-wisdom (*jñāna*) (Harshananda, 2008: Vol. 1, p. 627). On the other hand, according to the Jaina thoughts, there are four conditions of existence (*gati*) – hellish (*nāraka*), sub-human (*tiryañca*), human (*manuṣya*), celestial (*deva*) (Jaini, 1927: v. 146, p. 98). For one of these two reasons, certainly, for the first one, *gati* represents four.

Chronogram-words denoting five

According to Indian mythology, Kāmadeva is the god of sexual desire. He employs five fragrant flowers of the spring as the tips of his arrows. Those five flowers are *Aravinda* (day-lotus), *Aśoka* (the flower of Aśoka tree), *Cūta* (the flower of Mango tree), *Navamālikā* (jasmine), and *Nīlotpala* (blue lotus) (Mani, 1975: 378-379; Apte, 1893: 224 and 248; Cappeller, 1891: 40 and 174). For this reason, *śara* (arrow) refers to five. The chief instruments, with which a human being is equipped for acquiring knowledge from the external world and react to it, are the five *indriya*s (sense-organs). Those five *indriya*s are *rasanā* (gustatory perception system), *ghrāṇa* (olfactory system), *cakṣu* (ocular system), *śrotra* (auditory system), and *tvag* (somatosensory system) (Harshananda, 2008: Vol. 2, p. 68-69). Therefore, *indriya* (sense-organ) denotes five. There are five meanings of *artha*– meaning (*abhidheya*), wealth (*rai*), thing (*vastu*), purpose (*prayojana*), and retirement (*nivṛtti*) (Śāstrī, 1998: 453). According to the Vedānta philosophy, *prāpya* (that which is obtained, viz., God), *prāptṛ* (one who obtains, viz., the individual soul), *upāya* (the means of achieving), *prāptivirodhi* (that which obstruct attainment), and *prāpti* (attainment) are the five-truths (*artha-pañcaka*) that equip the aspirant of emancipation with the correct perspective (Harshananda, 2008: Vol. 1, p.161). For one of these two reasons, *artha* represents five.

Chronogram-words denoting six

Āyurveda recognizes six tastes. The *Aṣṭāṅga Hṛdaya* mentions them as *madhur* (sweet), *amla* (sour), *lavaṇa* (salty), *kaṭu* (bitter), *tikta* (pungent), and *kasāya* (astringent) (Vidyanath, 2013: v. 1.14, p. 13). They are called *ṣaḍrasa* or simply *rasa*. Hence, *rasa* refers to six. Unlike

many other countries, the Indian subcontinent has six seasons, namely, *vasanta* (spring), *grişma* (summer), *varşā* (rainy/monsoon), *śarad* (autumn), *hemanta* (fall winter), and *śiśira* (winter). Therefore, *ṛtu* (season) denotes six.

Chronogram-word denoting seven

According to the *Viṣṇupurāṇa*, there are seven mountains in the region of *Bharatavarṣa* – Mahendra, Malaya, Sahya, Śukti, Ṛkṣa, Vindhya, and Pāriyātra (Taylor, 2021: v. 2.3., p. 158; Caturvedī, 1998: 351). This is the reason why *adri* (mountain) denotes seven.

Table 2: Early chronogram-words

Chronogram-word	Number denoted	Inscription
candra	1	K. 323
indu	1	K. 323
bhujā	2	K. 323
dasra	2	K. 13
velā	2	C. 30 B 3
rāma	3	C. 74
gati	4	K. 756
pasāgi	4	Kebon-Kopi II
śruti	4	D. 4
vārinidhi	4	K. 13
artha	5	C. 74
indriya	5	K. 13, D. 4
śara	5	K. 13, K. 151
rasa	6	K. 13, D. 4
ṛtu	6	K. 13, K. 756
adri	7	C. 30 B 3
vasu	8	K. 323
kawihāji	8	Kebon-Kopi II
kha	0	K. 756, K. 151

Chronogram-words denoting eight

Vasu was the daughter of Dakṣa. Dharmadeva was her husband. Eight sons were born to her. They are known as *aṣtavasus* (eight *vasus*) (Mani, 1975: 65-66). Therefore, *vasu* denotes eight. *Kawihāji* (or *kawihāji*) is, according to Bosch, synonymous with *bhujaṅga* (snake). Vāsuki, Takṣa, Karkoṭaka, Śaṅkha, Gulika, Padma, Mahāpadma, and Ananta are eight-serpent-deities (*aṣṭanāga*) worshiped in Indian culture (Bosch, 1941: 50; Mani, 1975: 64). This is why *kawihāji* denotes eight.

Chronogram-word denoting zero

Since *kha* connotes hole, especially the hole in the nave of a wheel, aperture, especially an aperture of the human body as mouth, etc., hollow, organ of sense, void space, and sky (Cappeller, 1891: 143), it denotes zero.

3. The maxim *aṅkānāṃ vāmato gatiḥ*

aṅkānāṃ vāmato gatiḥ

Datta and Singh call it a rule and interpret it as follows.

> "The numerals proceed to the left (Datta and Singh, 1935: 62)."

M. D. Pandit calls it a dictum or rule and interprets it as follows.

> "The understanding ‹(*gati* from √*gam* 'to go', also 'to understand)› of the numbers ‹is to be done› in the reverse way ‹(*vamataḥ*)› (Pandit, 1993: 154)."

Sarma calls it a maxim and interprets it as follows.

> "The movement of the digits ‹is from the right› to the left (Sarma, 2009: 73)."

Prabowo et al. call it a principle (Prabowo et al., 2016). B. S. Shylaja and Seetharama Javagal call it a rule and interpret it as follows.

> "The numerals are to be read in the opposite direction – from right to left (Shylaja and Javagal, 2021: 212)."

As far as I know, Sarma is the first scholar who called it a maxim. I too would prefer to call it a maxim because a maxim is a rule that connects an action to its causes.

This maxim has been very popular among Indian mathematicians. Indian texts on astronomy and mathematics from their earliest times arrange the digits of a numerical quantity from right to left, beginning with the unit's place. Pandit also writes that,

> "Though this phrase [i.e., maxim] is very well-known and oft-stated and quoted in Sanskrit mathematics or in dealing with numbers outside the field of mathematics, it is nowhere explained or defined; not only this, but even its source is not traceable in any of the Sanskrit works, mathematical or non-mathematical (Pandit, 1992: 88)."

Interestingly, the maxim is found in the anonymous manuscript *Saṃjñā-nighaṇṭu* preserved at Asiatic Society of Mumbai (Sarma, 2009: 74-75). It is also found in the *Samayocitapadyamālikā*, a collection of phrases and proverbs to be quoted on appropriate occasion, as the fourth quarter. The whole verse is as follows.

> *aṅkeṣu śūnyavinyāsād*
> *vṛddhiḥ syāttu daśādhikā*।
> *tasmājjñeyā viśeṣeṇa*
> *aṅkānāṃ vāmato gatiḥ*॥
> (Dravid, 1993: v. 48, p. 5)

This "verse is quoted," writes Pandit, "as a joke on the number zero, which, in spite of having no value of its own, increases the value of number ten times when written or read with it. The source obviously seems to be secondary and its primary, mathematical source is nowhere traceable in any mathematical work, pre-Vedic, Vedic or post-Vedic. The dictum seems to have been handed down orally and not mentioned specifically anywhere (Pandit, 1993: 224-225)."

The manuscript *Saṃjñā-nighaṇṭu* also does not seem to be a work on mathematics as *nighaṇṭu* is a Sanskrit term for a traditional collection of words usually accompanied by brief annotations. Therefore, we can say that not much is known about the origin and antiquity of this Indian maxim.

Haribhadra Sūri of the eighth century refers to

> "*gaṇiaṃ saṃkhāṇaṃ suṃdarīi vāmeṇa uva^iṭṭhaṃ*"

in his commentary (*vṛtti*) on the *Āvaśyakaniryukti* (Mahārāja, 1980/1981: v. 13, commentary on v. 207, p. 88). The gist of this citation is that Sundarī learned to understand numbers in the order of their digits from right to left from her father Lord Ṛṣabhadeva, the first Tīrthaṅkara in Jainism. This story, at least, makes sure that the antiquity of the maxim dates back to before Haribhadra.

S. R. Sarma seems to be practical in saying that the maxim must have originated at about the same time as the place-value (Sarma, 2009: 74). As to the date of invention of the place-value system in India, Datta and Singh write,

> "The place-value system was known in India about 200 B.C.… It is possible that further evidence may force us to fix an earlier date (Datta and Singh, 1935: 87-88)."

Modern scholars of ancient Indian mathematics and Southeast Asian chronograms often discuss the maxim with the word-symbol system. For example, Shylaja and Javagal refer to the maxim to explain how to read a given word-symbol notation (Shylaja and Javagal, 2021: 212). Not only this, the word-symbol notations written in the sinistral move are said to have followed the maxim and the chronogram written in the dextral move is said to have opposed the maxim (Sarma, 2009: 71; 2012: 40; Prabowo et al., 2016: 5084). These presentations of following and opposing the maxim, as far as I know, do not reflect in any traditional source, whether from India or Southeast Asia. But it is also true that in both India and Southeast Asia, preference was given to the sinistral move. Datta and Singh write,

> "No explanation as to why the right to left arrangement was preferred in the word [i.e., word-symbol] system is to be found in any of the ancient works. The following explanation suggests itself to us, and we believe that it is not far from the truth: [1] The different words forming a number chronogram were to be so selected that the resulting word expression would fit in with the metre used. … [2] The right to left arrangement is thus due to the desire of the

> mathematicians to look upon the process of formation of the word-chronogram [i.e., word-symbol notation] as a sort of arithmetical operation (Datta and Singh, 1935: 62)."

With regard to arithmetical operation Sarma says,

> "What they [i.e., Datta and Singh] mean by 'arithmetical operation' is the following. In addition, subtraction and multiplication, we usually commence the operation at the unit's place and then proceed to higher powers in the right-to-left direction (Sarma, 2009: 74)."

But we have already seen that the higher powers (i.e., higher place-values) in the word-symbol notations written in the sinistral move proceed from left to right, not in the right-to-left direction. If "arithmetical operation" had been the reason for preferring the sinistral move, the dextral move would have been the best option. However, the above two arguments, except for the involvement of the notion of "arithmetical operation," are valid not only for the sinistral move, but also for the dextral move.

Now we would like to know the rationale behind the maxim. We can know it through a very valuable paper written in Bengali by B. B. Datta under the title *Aṅkānāṃ vāmato gatiḥ*: *Gaṇitavidhi*, published in *Sāhitya Pariṣat Patrikā 2: 70-80* in the year 1930-1931. He discussed it lucidly, citing, among others, two passages from Gaṇeśa Daivajña (1545 CE) and Nṛsiṃha Daivajña (1621 CE). Apart from these two savants, one more important savant is Kṛṣṇa Daivajña (c. 1600 CE), whose passage Datta does not refer to and whom Nṛsiṃha himself refers to on this maxim. Their three passages are in Sanskrit. By reproducing them in full, Sarma translates them into English quite literally (Sarma, 2009: 76-84). He briefly describes the explanation offered by these three savants for the rationale of the maxim as follows.

> "The identification of a notational place as the ten's place or the hundred's place is possible only with reference to the unit's place; that is to say, only when we proceed in the right-to-left direction and note that each notational place is ten's times higher than the previous notational place. We cannot start from the upper limit and proceed to the right, …, for the upper limit is uncertain. Numbers being infinite, there cannot be any upper limit. On the other hand, the lower limit, i.e., the unit's place, is certain. In other words, this arrangement is intrinsically connected with the principle of the decimal place-value (Sarma, 2009: 75)."

As we have seen above, the interpretation given by Pandit to the maxim is quite different from the ones given by others. The careful investigations and considerations that led him to make such an interpretation are as follows.

> "Veda is composed," writes he, "in language and not in number-symbols [i.e., 1, 2, 3, …, 9]. We will not find, therefore, any symbol for any number, …, in it. … At this stage, we can see that the Vedic language can be classified into two types; One, that language or words which describe and analyse the non-mathematical facts, like the praise of the gods etc. … We call this type of language as non-mathematical language. The second type of language which is used is full of words for mathematical numbers. … We may call this language as mathematical language. … Thus in the passage [from the *Ṛgveda*], *dvādaśa pradhayaḥ, cakram ekam, trīṇi nabhyāni, ka u tac ciketa; tasmin sākam triśatā na śaṅkavaḥ arpitāḥ ṣaṣṭir na calācalāsaḥ*, R.V. 1.164.48, the words *dvādaśa*, *ekam*, *trīṇi*, *triśatā*, *ṣaṣṭiḥ* are mathematical language, giving out mathematical information, while all others are non-mathematical language, giving out non-mathematical information. … Since both the types are languages, they are to be understood properly. … [In] the non-mathematical language, say a word like *vajra–bhṛt*, is understood in the order in which the words are set therein. … But it is

not so in the case of the mathematical language, say a word *dvādaśa* in which traditionally *daśa* [i.e., ten] is understood first and then comes the turn of the word *dvā* [i.e., two]. The two types of languages, though composed of the same Sanskrit phonemes, morphemes and words, require different ways of understanding. … It is in order to avoid the confusion between the two [languages] that the Sanskrit mathematicians in Vedic times have spelled out a dictum … *aṅkānāṃ vāmato gatiḥ*, … The word *aṅka* in the verse [from the *Samayocitapadyamālikā*] refers to the number-symbols for the numbers are to be written down or read or understood first in the order in which they are spoken and then the order is to be reversed. Thus, the word *dvā-daśa* is to be first understood in symbols as 2-1 and then the figures 2 and 1 are to be read in reverse position as 1-2 and the number will be written as 12. … The rule *aṅkānāṃ vāmato gatiḥ* is, therefore, of utmost importance in understanding the numbers. The rule actually seems to have been spelled out to understand the Vedic texts as principle of interpretation of Vedic language and seems to have been coined out of necessity. But, surprisingly enough, later post-Vedic writers adhered to this rule strictly and wrote the numbers purposefully and unnecessarily in the reverse fashion i.e. from right to left; … [for example, the partial word-symbol notation in the] *Sūryasiddhānta*: *nava-vasu-sapta-aṣṭa-kha-nava-aśva* = 9-8-7-8-0-9-2, which is = 2908789 [when reversed]. … The word *vāma* [in the *aṅkānāṃ vāmato gatiḥ*] is peculiar here. It signifies the sense of 'left, or reverse' in classical Sanskrit. … *Vāma* in the sense of 'reverse' indicates or refers to the direction opposite to the one in which one is proceeding. … The *devanāgarī* [script] is written from left to right; the *vāma* of this direction will be from right to left. … The only purpose that seems to be implied behind the above dictum is that the principle aims at laying down the order of the number-symbols while writing them down. This leads us to the next condition that writing of the numbers at least in symbols was prevalent in the Vedic and the immediately succeeding post-Vedic times. Without such a presumption, the dictum *aṅkānāṃ vāmato gatiḥ* seems to be meaningless. Though the principle seems to have been enunciated later, it was applicable in Vedic times also (Pandit, 1993: 152-156)."

The interpretation given by Shylaja and Javagal for the maxim, due to their taking the term *vāma* to mean "opposite direction," is close to that given by Pandit, but we have already seen the purpose for which they refer to the maxim.

From the entire discussion above it becomes clear that the two moves are independent of the maxim.

If the dextral move had been undesirable or contrary to the maxim, it would have been discarded in the long run. But we are able to see through the previous sections and subsections that it was continued to be used for writing word-symbol notations in India and for writing chronograms in Southeast Asia. In India, Nemicandra used both of the moves in his works not only to write word-symbol notations but also for composing *kaṭapayādi* notations (Jaini, 1927: v. 125, p. 88; Mukhtara and Patni, 1975: v. 21, p. 29; Jadhav, 1998a: 51-52). In addition to what we have seen in this paper, many other chronograms were written in the dextral move in Southeast Asia (Annisa, 2011).

The only way for a reader to know what move was used to write a given word-symbol notation or chronogram is to understanding the entire text, which contains the given word-symbol notation or chronogram, and its various aspects. This approach was also adopted by Bosch. The reason which he offers for the chronogram '*kawihāji-pañca-pasāgi*' to have been written in the dextral move is the following. The Śaka year decoded from this chronogram is 854 or, with the usual conversion of the number, 458. 458 is much too early for the type of writing used, but 854 corresponds well with it. Therefore, 854 is undoubtedly the correct reading (Bosch, 1941: 50). Historian Hasan Djafar supports reading this *sengkala* as

854 and says that this reading matches the actual historical facts (Prabowo et al., 2016: 5083).

4. On two points from the quintet's paper

The quintet writes,

> "The discussion about the mathematical procedures in the numbers' arrangement of Candrasengkala was not given adequate proportion. [The] available resources [i.e., studies] will not be enough to provide a complete picture of Candrasengkala as it falls in the category of the intersection of mathematics, anthropology and history; or what is usually called as Ethnomathematics (Apsari et al., 2021: 2)."

This is the first point I would like to comment on. What the quintet took here as a mathematical procedure is the arrangement of "numbers" in *candrasengkala*. *Sengkala* is a number in itself. It is usually made up of digits, but not necessarily; for example, *surya* (i.e., *ina*) denotes twelve. These digits can be called "numbers" when spoken with their place-values, otherwise not.

We are able to see that the quintet recognized "*sengkalas*" or, to be more precise, "the mathematical procedures and ideas employed in writing *sengkalas*" as ethnomathematics.

Since the practice of *sengkalas* or chronograms written using word-symbols has not been confined to a specific cultural group, they do not match the following definition given by D'Ambrosio.

> "We will call *ethnomathematics* the mathematics which is practiced among identifiable cultural groups, such as national-tribal societies, labor groups, children of a certain age bracket, professional classes, and so on (D'Ambrosio, 1985: 45)."

The practice of chronograms has been widespread in the Javanese-Balinese tradition of Indonesia. We have already seen that this practice is well preserved in the rich Indonesian literature and archaeological remains. In Indonesia itself, the practice has a rich history dating back centuries. What is more, the practice also has a rich history in Southeast Asian countries like Cambodia and Vietnam. In addition, India, where the practice came from, also has a centuries-old rich history of word-symbol notations and their use in various fields of learning. In such a rich scenario, we cannot recognize chronograms or "the mathematical ideas employed in writing them" as ethnomathematics. Instead, we can look at them under the history of mathematics.

The second point relates to the quintet's approach to cultural-based mathematics education through chronograms. The commutative property holds true for addition: $x + y = y + x$ where x and y are two numbers. The quintet takes, among others, one chronogram *śruti-indriya-rasa* from Inscription "D. 4" as a tool to aid in the actual learning and teaching of this property. All of this can also be used to illustrate how indigenous peoples had an understanding of the property, says the quintet. The quintet's interpretation is that the year depicted in this chronogram is written in ascending order (i.e., $4\times10^0 + 5\times10^1 + 6\times10^2$) rather than in descending order (i.e., $6\times10^2 + 5\times10^1 + 4\times10^0$). The quintet further says that the descending order is a common order (Apsariet al., 2021: 1 and 6).

The pre-requisite for this innovation is that the learners must be aware of the concept of place-value. Chronograms are only in three digits and four digits, not in two digits. We have seen above that the number of the additive arrangements of two numbers is 2 (= 2!). Before seeing how the commutative property works for more than two numbers, I would like to say that the quintet decodes *śruti*, *indriya*, and *rasa* with their place-values. Therefore, the quintet has

4×10^0 (= a), 5×10^1 (= b), and 6×10^2 (= c)

respectively. The additive arrangements of three numbers will be 6 (= 3!) as shown below.

$$a+b+c = a+c+b = b+a+c = b+c+a = c+a+b = c+b+a$$

The quintet took the first and the last, which they called in ascending and descending orders respectively, because the rest were not practiced in forming chronograms; nor were they easy to use. Their ascending and descending orders are the sinistral and dextral moves respectively. Therefore, their innovation deserves praise.

V. CONCLUSION

Each of the nineteen early chronogram-words from Cambodia, Vietnam and Indonesia is related to shape, body parts, season, oceanic phenomenon or ancient Indian astronomy or is a bearer of deeply rooted thought developed in ancient Indian society from the Vedic, *paurāṇika*, epic, philosophical or *Āyurvedika* culture. Because of denoting the year in *Śaka* calendar, the word-symbol notation, which was known by various names in India and used for various purposes, received the name *sengkala* in Indonesia.

More such studies on Southeast Asian chronograms may shed more light on the mathematical considerations used in writing them, such as through this study we have found the following. The terms used for word-symbols in India are component-oriented while the term *sengkala* is application-oriented. Both the sinistral and dextral moves used for writing word-symbol notation in India and chronogram in Southeast Asia were independent of the maxim *aṅkānāṃ vāmato gatiḥ*.

The teaching and learning of mathematics can be made culturally relevant using chronograms. This type of teaching can help students solidify their mathematical concepts.

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Abbreviations and notations

AJ Anno Javanica

ŚaE Śaka Era. This is generally 78 years behind of Current Era, except during January to March when it is behind by 77 years. Śaka year starts from day 1 of month Caitra. This date generally falls in the month of March or April.

Skt. Sanskrit. The term(s) put by me after Skt. are intended to aid the reader to understand those terms preceding it through Sanskrit equivalents.

[…] A pair of square brackets, wherever used, contains a paraphrase inserted by me to achieve comprehensiveness together with clarity.

‹…› A pair of pointing angle quotation marks, wherever used, contains what Bosch, Petit, Pandit, or Sarma himself inserted.

Appendix: Word-symbols from Indian sources and other Javanese chronogram-words

We have compiled the "word-symbols from Indian sources" and "other chronogram-words" referred to in this paper in Table 3 and Table 4 respectively. We will explore their significance in this appendix.

Sasi (Skt. *śaśi*) and *vidhu* are the synonyms of *candra*. We have already explored in subsection 2 of section IV why *candra* denotes one. The earth is unique in many respects. Therefore, *bumi* (Skt. *bhūmi*, earth) represents one. Since *tunggal* means single or united, it refers to one.

The reason why *dasra* signifies two applies to each of *aśvi* and *aśva*. *Loyaṇa* (Skt. *locana*) means eye. *Ṇayaṇa* (Skt. *nayana*) is its synonym. Since every human being has two eyes, each of these two words denotes two.

Fire has three properties – to heat, burn, and illuminate. This view seems to be held by Javanese in modern times (Wijayatno, 2003). In Vedic thought, fire is considered as the mouth of the gods and the goddesses. The sun in space, lightning in the environment, and fire on land

show its presence at three levels (Lochtefeld, 2002: 14–15). There are three sacrificial fires known as *gārhapatyāgni* (the house holder's fire), *āhavanīyāgni* (the fire situated in the east and used as the main offering fire), and *dakṣiṇāgni* (the fire situated in the south and used for certain rituals) (Rangacharya, 1912, p. 287). For one of these three reasons, certainly, for the last and third one, each of *agni* (fire) and its synonym *dahana* represents three. According to the Indian philosophical system, *guṇa*, meaning quality or attribute, refers to three basic components of nature (*prakṛti*) –*sattva*, *rajas*, and *tamas*. *Sattva* is responsible for knowledge and pleasure. *Rajas* is accountable for activity and passions. *Tamas* gives rise to indolence, sleep and evil (Śāstrī, 1998: 47; Harshananda, 2008: Vol. 1, p. 667). This is the reason why *guṇa* denotes three. According to the *paurāṇika* thought, Lord Śiva had three eyes. *Hara* is his epithet. Hence *hara-nayaṇa* (Skt. *hara-nayana*) (i.e., Śiva's [three] eye[s] or Śiva's [third] eye) represents three.

Abdhi and *samudra* are the synonyms of *vārinidhi*. We have already explored why *vārinidhi* represents four. *Yuga* is a unit of the length of time as evolved in ancient India. There are four *yugas* – *kṛtayuga*, *tretāyuga*, *dvāparayuga*, and *kaliyuga*. Therefore, *juga* (Skt. *yuga*) represents four. The reason why *śruti* indicates four applies to *veda* also. The term *kerta* appears to be related to *kṛta* in Sanskrit and we have already explored the significance of *kṛta* in the early part of subsection 1 of section IV.

The term *iṃdiya* appearing in Table 3 is the Prakrit spelling of *indriya*. We have already seen in subsection 2 of section IV why *indriya* refers to five. *Śara* appearing in both Table 3 and Table 4 has already appeared in Table 2. *Bāṇa* is its synonym.

According to Jaina ontology, the numerous categories of beings (*jīvas*) are grouped into six main ones. *Khara* is linked to these six categories. This is why *khara* denotes six (Jadhav, 2019: 12). Prior to Nemicandra (c. 981 CE), Mahāvīra (c. 850 CE) included it in his versified list of word-symbols to denote six (Rangacharya, 1912: vv. 1.58, pp. 7 and 289). *Rasa* appearing in both Table 3 and Table 4 has already come in Table 2. We have already seen why *rttu* (Skt. *ṛtu*) represents six.

Muṇi (Skt. *muni*), meaning ascetics, is the synonym of *ṛṣī*. *Aṅgirasaṛṣis* are seven in number. By way of the *paurāṇika* tradition they have reached us as *saptarṣi* (seven sages [or seven stars of the Great Bear]) (Jadhav, 1998b: 3). This is the reason why *muṇi* denotes seven. *Ṇaga* (Skt. *naga*) and *śail* each are the synonym of *adri*.

Hatthī (Skt. *hastin*) means elephant. According to the *paurāṇika* thought, Airāvat in *pūrva* (east), Puṇḍarīka in *āgneya* (south-east), Vāmana in *dakṣiṇa* (south), Kumuda in *naiṛta* (south-west), Añjana in *paścima* (west), Puṣpadanta in *vāyavya* (north-west), Sārvabhauma in *uttara* (north), and Supratīka in *īśāna* (north-east) are the eight elephants who hold the earth in the respective eight directions (Śāstrī, 1998, p. 25). This is the reason why *hatthī* represents eight. We have already seen why *vasu* denotes eight. *Nāga*, which is written in Indonesian as *naga*, is the synonym of *bhujaṅga* (Annisa, 2011: 96; Listya and Pratama, 2019: 35 and 38-39; Wijayatno 2003). We have seen in subsection 2 of section IV why *bhujaṅga* (snake) represents eight. That is why *naga* (Skt. *nāga*) in "*dwi naga rasa tunggal*" denotes eight.

Therefore, it should be noted that *naga* in Sanskrit is different from *naga* in Indonesian. The former means mountain whereas the latter snake. Annisa's attention seems not to have gone towards this when she decoded *muni-śaśi-naga* given in the inscription of Pratihāra Vatsarāja as 817 ŚaE. She identified *naga* as *nāga* (*ular*, meaning snake in Indonesian) (Annisa, 2011: 6). We have already seen in subsection 1 of section IV that *muni-śaśi-naga* was decoded as 717 ŚaE.

According to the Jainas, there are nine *balabhadras* (gentle heroes) of the present *avasarpiṇī* (descending half cycle of the cosmic time) – Vijaya, Acala, Bhadra, Suprabha, Sudarśana, Nandisena, Nandimitra, Rāma, and Balarāma (Jadhav, 2019: 2). For this reason, *bala* (i.e., *balabhadra*) denotes nine. As per the *paurāṇika* texts, *Sūrya* (Sun), *Candra* (Moon),

Śukra (Venus), *Budha* (Mercury), *Kaja* or *Maṅgala* (Mars), *Bṛhaspati* (Jupiter), *Śani* (Saturn), *Rāhu* (ascending lunar node), and *Ketu* (descending lunar node) are the *navagrahas* (nine planets) (Mani, 1975: 295–297). Therefore, *gaha* (Skt. *graha*) denotes nine. Since there were nine kings in the Nanda dynasty, who ruled in the northern part of the Indian subcontinent, known as Magadha, during the pre-Christian Era (Limaye, 2019), *nanda* denotes nine. *Ddhi* (Skt. *ṛddhi*) means spiritual attainment or supernatural prosperity. I could not explore why Nemicandra (c. 981 CE) employed it to denote nine. According to the ancient Indian thought, *mahāpadma*, *padma*, *śaṅkha*, *makara*, *kacchapa*, *mukunda*, *kunda*, *nīla*, and *kharva* are *navanidhi* (nine treasures) (Mani, 1975: 544; Rangacharya, 1912: 291). Therefore, *ṇidhi* (Skt. *nidhi*) denotes nine.

Ravi, *Sūrya*, and *Ina* are the names of the Sun. According to the *paurāṇika* thought, the Sun was born to Aditi. She was the wife of Kaśyapa. Several sons were born to her. They are known by the names Ādityas, etc. Of these, Ādityas, (the sons of Aditi) are twelve in number. According to some writers, the concept of twelve Ādityas corresponds to twelve months (Mani, 1975: 770; Limaye, 2019). This is the reason why *ravi*, *sūrya*, and *ina* each denote 12.

According to ancient Indian astronomy, the lunar mansion is the name of $1/27^{th}$ part of the path of the moon round the earth. For this reason, *bha* (lunar mansion) denotes twenty-seven. Earlier there were twenty-eight lunar mansions (Harshananda, 2008: Vol. 2, p. 361).

Since most adults have thirty-two teeth, *danta* (tooth) denotes 32.

We have already seen why *kha* refers to zero. Since sky is empty, *ṇabha* (Skt. *nabha*, sky) represents zero. Since *ilang* and *sirna* means "lost" and "disappeared" respectively, they each refer to zero.

Table 3: From Indian sources

Word-symbol	Denoted
sasi, *vidhu*	1
aśvi, *aśva,loyaṇa*, *ṇayaṇa*	2
agni, *dahana*, *guṇa*, *hara-nayaṇa*	3
abdhi, *juga*, *samudra*, *veda*	4
iṃdiya, *bāṇa*, *śara*	5
khara, *rasa*	6
muṇi, *ṇaga*, *śail*	7
hatthī, *vasu*	8
bala, *gaha*, *nanda*, *ddhi*, *ṇidhi*	9
ravi, *sūrya*	12
bha	27
danta	32
kha, *ṇabha*	0

Table 4: Other chronogram-words

chronogram-word	Denoted
bumi, *tunggal*	1
kerta	4
śara	5
rasa, *rttu*	6
naga	8
ina	12
ilang, *sirna*	0

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